Uncertainty of hyperon couplings and the electrochemical potential in neutron star matter

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Uncertainty of the hyperon couplings, in particular, that of the Σ^-, in dense matter raises the question of the behavior of the electrochemical potential in neutron star matter, which is crucial to the possible presence of the kaon condensed phase. We show that regardless of this uncertainty, the Λ hyperon, whose coupling can be constrained by its binding in nuclear matter and other observations, by itself, or also aided by the Ξ^-, introduces a saturation of the electrochemical potential just as the Σ^- would otherwise do, which tends to mitigate against kaon condensation. The maximum possible mass of neutron stars appears to be $\sim(1.5\text{–}1.7)M_\odot$ independent of the uncertainties, the limit being imposed by any one of hyperonization, deconfinement, or kaon condensation. Interestingly, such a limit is barely higher than the Chandrasekhar limit on the iron core mass of presupernova stars. This leaves a very small mass window for the existence of neutron stars, for the occurrence of supernovas, and therefore for a universe containing heavy elements and at least one planet with life.

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I. INTRODUCTION

Hyperons, the strange members of the baryon octet, are likely to exist in high density matter and, in particular, in its charge neutral form, often referred to as neutron star matter. The Pauli principle practically assures that their presence will lower the Fermi energy of baryon species and hence the total energy at given baryon density. However, aside from this general argument, the coupling constants of hyperons also influence the extent of their participation. The Λ hyperon is a partial exception to this uncertainty [1]. Its couplings can be at least constrained by the experimentally extrapolated value of its binding in nuclear matter [2], by the results of an analysis of hypernuclear levels [3], and by the requirement that theory account for neutron stars of mass as great as $1.5M_\odot$.

The important neutron star properties that hyperons effect are the limiting neutron star mass and the possibility of kaon condensation. As compared to models populated only by nucleons and leptons, hyperons reduce the maximum mass by as much as $3/4M_\odot$ [1]. The reason their presence strongly effects the possibility of kaon condensation is as follows [4]: The effective mass of kaons in nuclear matter is reduced from its vacuum mass by an attractive interaction with the nuclear medium [5]. If the K^- effective mass sinks to a value of the electron chemical potential as the density of matter increases, the K^- can thereafter replace the electron as the charge neutralizing agent in neutron star matter. The kaons can all condense in the zero momentum state, whereas electrons have to occupy ever higher momentum states with increasing density. *However*, hyperons may saturate the electron chemical potential at a relatively low density, either postponing the appearance of a kaon condensate to a high density or preempting it altogether. The reason that hyperons can do this is because they carry conserved baryon charge and they occur in all three charge states ± 1 and 0. Therefore it may happen that charge neutrality can be achieved most energetically favorably among baryons with little participation of leptons. (Lepton number is not conserved because of neutrino loss from the star.) The foregoing conclusions of Ref. [4] have been confirmed in subsequent work [6–9].

That hyperons can contribute to the saturation of the electron chemical potential and may preempt thereby the condensation of kaons depends, at first sight, on the Σ^- since it is the lowest mass baryon of negative charge and can replace a neutron and electron. Extrapolated atomic data suggest that it may feel repulsion at high density, which would mitigate against its appearance in dense matter, although this remains inconclusive [10]. Indeed, it has been suggested that the absence of the Σ^- would mitigate the negative effect that hyperons have on kaon condensation [11–13].

However, we show in this paper that even if the Σ^- is totally absent from dense neutral matter, the Λ hyperon, by itself or aided by the Ξ^-, also causes the electron chemical potential to saturate and then decrease with increasing density. The Λ is known to experience an attractive potential in normal nuclear matter [2] as does the Ξ^- [14–16]. The Λ can replace neutrons at the top of their Fermi sea with a reduction in the high value of the three-component of the isospin of neutron star matter, thus reducing the asymmetry energy and with no increase in electron and proton population with increase of density [4]. The Ξ^- can replace a neutron and electron and also enhance the proton population at the expense of the neutron, just as the Σ^-, and has a low density threshold in the absence of the Σ^-. The net effect is that hyperons disfavor kaon condensation by terminating the growth of the electron population and electrochemical potential with increasing density, even if the Σ^- interaction were so strongly repulsive that it is absent from neutron star matter in the density range relevant to those stars.

II. THEORY

We describe nuclear matter by the mean field solution of the covariant Lagrangian [4,17–22] which is a generalization of the model introduced first by Johnson and Teller [23], by

Duerr [24], and later by Walecka [25]:

$$\begin{aligned}\mathcal{L}=\sum_B \bar{\psi}_B\bigg(& i\gamma_\mu\partial^\mu - m_B + g_{\sigma B}\sigma - g_{\omega B}\gamma_\mu\omega^\mu \\ & -\frac{1}{2}g_{\rho B}\gamma_\mu\boldsymbol{\tau}\cdot\boldsymbol{\rho}^\mu\bigg)\psi_B + \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2) - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} \\ & + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\boldsymbol{\rho}_{\mu\nu}\cdot\boldsymbol{\rho}^{\mu\nu} + \frac{1}{2}m_\rho^2\boldsymbol{\rho}_\mu\cdot\boldsymbol{\rho}^\mu - \frac{1}{3}bm_n(g_\sigma\sigma)^3 \\ & - \frac{1}{4}c(g_\sigma\sigma)^4 + \sum_{e^-,\mu^-}\bar{\psi}_\lambda(i\gamma_\mu\partial^\mu - m_\lambda)\psi_\lambda. \end{aligned} \tag{1}$$

The advantage of the model as compared with other models of nuclear matter is that it can be made to agree with five nuclear properties at *saturation* density, the highest density for which we have any empirical knowledge, and it extrapolates causally to all densities. The baryon species, denoted by B, are coupled to the scalar, vector, and vector-isovector mesons σ, ω, and $\boldsymbol{\rho}$. The masses are denoted by m with an appropriate subscript. The sum on B is over all the charge states of the lowest baryon octet ($p,n,\Lambda,\Sigma^+,\Sigma^-,\Sigma^0,\Xi^-,\Xi^0$) as well as the Δ quartet and the triply strange baryon Ω^-. However, the latter two are not populated up to the highest density in neutron stars, nor are any other baryon states save those of the lowest octet for reasons given elsewhere [4]. The cubic and quartic σ terms were first introduced by Boguta and Bodmer so as to bring two additional nuclear matter properties under control [19]. The last term represents the free lepton Lagrangians. How the theory can be solved in the mean field approximation for the ground state of charge neutral matter in general beta equilibrium (neutron star matter) is described fully in Refs. [4,17].

The mean values of the nonvanishing meson fields are denoted by $\sigma,\omega_0,\rho_{03}$, in which case the baryon effective masses are given by $m_B^\star = m_B - g_{\sigma B}\sigma$ and the baryon eigenvalues by

$$e_B(k) = g_{\omega B}\omega_0 + g_{\rho B}\rho_{03}I_{3B} + \sqrt{k^2 + m_B^{\star 2}}. \tag{2}$$

In the above equations, I_{3B} is the isospin projection of baryon charge state B.

The Fermi momenta for the baryons are the positive real solutions of

$$e_B(k_B) = \mu_B \equiv b_B\mu_n - q_B\mu_e, \tag{3}$$

where b_B and q_B are the baryon and electric charge numbers of the baryon state B, and μ_n and μ_e are independent chemical potentials for unit baryon number and unit negative electric charge number (neutron and electron, respectively). The lepton Fermi momenta are the positive real solutions of

$$\sqrt{k_e^2 + m_e^2} = \mu_e, \qquad \sqrt{k_\mu^2 + m_\mu^2} = \mu_\mu = \mu_e. \tag{4}$$

These equations (3) and (4) ensure chemical equilibrium.

Charge neutrality is expressed as identically vanishing charge density

$$q \equiv \sum_B (2J_B+1)q_B k_B^3/(6\pi^2) - \sum_\lambda k_\lambda^3/(3\pi^2) = 0, \tag{5}$$

where the first sum is over the baryons whose Fermi momenta are k_B and the second sum is over the leptons e^- and μ^-. By simultaneously solving the meson field equations, the condition for charge neutrality, and the conditions for chemical equilibrium (3), (4), we get the solution for the three mean fields, the two chemical potentials, the two lepton Fermi momenta, and the N baryon Fermi momenta (where N is the number of baryon charge states populated) of beta-stable charge-neutral matter called neutron star matter at the chosen baryon density in the hadronic phase,

$$\rho = \sum_B (2J_B+1)k_B^3/(6\pi^2). \tag{6}$$

The equation of state can be calculated at each baryon density for which the solution for the $7+N$ variables enumerated above has been found. It is

$$\begin{aligned}\epsilon = {} & \tfrac{1}{2}m_\sigma^2\sigma^2 + \tfrac{1}{2}m_\omega^2\omega_0^2 + \tfrac{1}{2}m_\rho^2\rho_{03}^2 \\ & + \sum_B \frac{2J_B+1}{2\pi^2}\int_0^{k_B}\sqrt{k^2+m_B^{\star 2}}\,k^2dk \\ & + \sum_\lambda \frac{1}{\pi^2}\int_0^{k_\lambda}\sqrt{k^2+m_\lambda^2}\,k^2dk, \end{aligned} \tag{7}$$

which is the energy density, while the pressure is given by

$$\begin{aligned} p = {} & -\tfrac{1}{2}m_\sigma^2\sigma^2 + \tfrac{1}{2}m_\omega^2\omega_0^2 + \tfrac{1}{2}m_\rho^2\rho_{03}^2 \\ & + \frac{1}{3}\sum_B \frac{2J_B+1}{2\pi^2}\int_0^{k_B} k^4\,dk/\sqrt{k^2+m_B^{\star 2}} \\ & + \frac{1}{3}\sum_\lambda \frac{1}{\pi^2}\int_0^{k_\lambda} k^4dk/\sqrt{k^2+m_\lambda^2}. \end{aligned} \tag{8}$$

These are the diagonal components of the stress-energy tensor

$$\mathcal{T}^{\mu\nu} = -g^{\mu\nu}\mathcal{L} + \sum_\phi \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\partial^\nu\phi. \tag{9}$$

Five of the constants of the theory can be algebraically determined by the properties of nuclear matter [17]. The constants are the nucleon couplings to the scalar, vector, and vector-isovector mesons g_σ/m_σ, g_ω/m_ω, and g_ρ/m_ρ and the scalar self-interactions defined by b and c. The nuclear properties that define their values used here are the binding energy $B/A = -16.3$ MeV, baryon density $\rho = 0.153$ fm^{-3}, symmetry energy coefficient $a_{sym} = 32.5$ MeV, compression modulus $K = 240$ MeV, and nucleon effective mass $m^*/m = 0.78$. How these choices are related to empirical data is discussed in Chap. 4, Sec. 5, of Ref. [17].

Nuclear matter at normal density does not depend on the hyperon couplings. Elsewhere we have shown how they can be made consistent with (1) the data on hypernuclear levels,

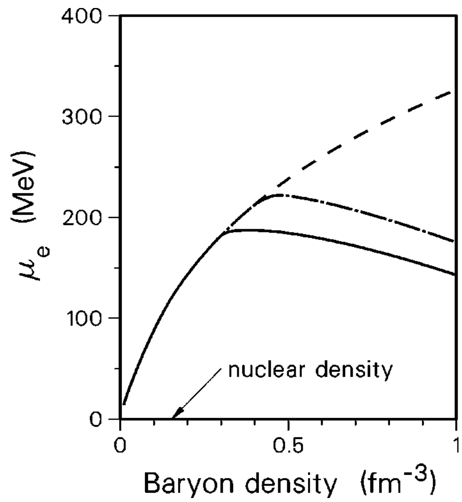

FIG. 1. Electrochemical potential in neutron star matter as a function of density. Three cases are compared: (1) only nucleons and leptons are present (dashed line), (2) nucleons, hyperons, and leptons are present (solid line), (3) nucleons, leptons, and hyperons except the Σ^- are present (dash-dot line).

(2) the binding of the Λ in nuclear matter (which can be determined quite accurately from an extrapolation of the hypernuclear levels to large atomic number A), and (3) neutron star masses [1]. We shall assume that all hyperons in the octet have the same coupling as the Λ. The couplings are expressed as a ratio to the above-mentioned nucleon couplings,

$$x_\sigma = g_{H\sigma}/g_\sigma, \qquad x_\omega = g_{H\omega}/g_\omega, \qquad x_\rho = g_{H\rho}/g_\rho. \quad (10)$$

The first two are related to the Λ binding by a relation derived in [1] and the third can be taken equal to the second by invoking vector dominance. Together the hyperon couplings are limited to the range $0.5 < x_\sigma < 0.7$ [1] and we take $x_\sigma = 0.6$. The corresponding value of x_ω is 0.658.

III. RESULTS

To illustrate that the behavior of the electrochemical potential is only slightly influenced by the question of whether the Σ^- hyperon experiences a strong repulsion in nuclear matter, we consider two cases, in one of which all hyperons are coupled with the same strength as the Λ, whose coupling can be constrained by observation as described above. In the other case, we consider the extreme case where the Σ^- experiences such a strong repulsion that it does not appear at all in matter to densities exceeding those found in neutron stars. To illustrate how hyperons arrest the growth of the electrochemical potential with increasing density, we compare the above cases with a model in which only nucleons and leptons appear. In the latter case, the electrochemical potential increases monotonically with density, and it is on that behavior that the case for kaon condensation mainly rests. The results can be compared in Fig. 1.

It is apparent that the hyperons limit the growth of the electrochemical potential at a density of 2.5–3 times nuclear density and bring about its monotonic decrease at higher density from a maximum value of about 200 MeV, which is far below the vacuum mass of the K^- of 494 MeV. This renders kaon condensation problematic, and further progress on this question will require very accurate evaluation of the behavior of the K^- mass as a function of density, as well as continuing experimental work on hyperon interactions.

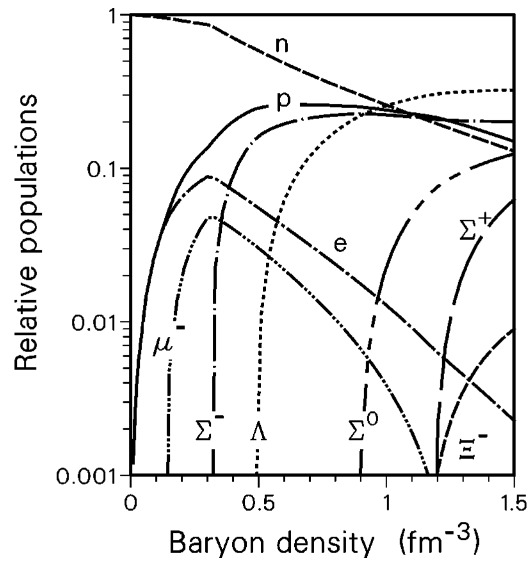

FIG. 2. Particle populations in neutron star matter containing nucleons, hyperons, and leptons.

It is interesting to see how the hyperon populations adjust to the possible absence of the Σ^-. This can be viewed by comparing Figs. 2 and 3. The second of these two figures is the one in which the Σ^- is absent. We see that to compensate the absence of the Σ^-, the Λ threshold has been reduced somewhat, and the Ξ^- threshold has been greatly reduced. These changes take place to most economically bring about charge neutrality in neutron star matter and illustrate how

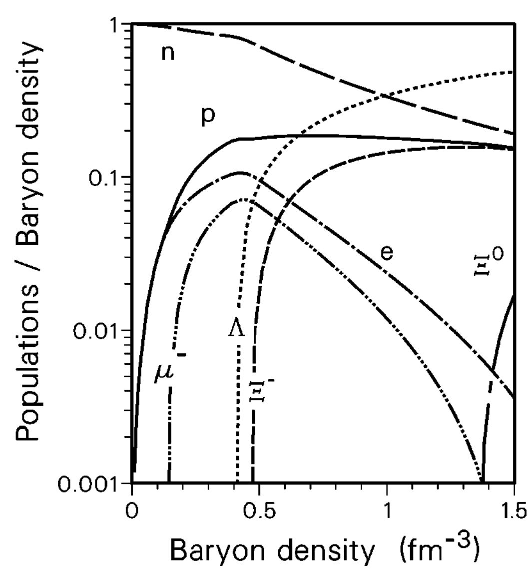

FIG. 3. Particle populations in neutron star matter containing nucleons, hyperons (absent the Σ^- because of strong repulsion), and leptons.

powerful the Pauli principle is in arranging fermion populations of conserved type in dense matter so as to minimize energy at given density. The great reduction of the Ξ^- threshold in the absence of the Σ^- occurs because it is charge favored, replacing a neutron and electron at the top of their Fermi seas (although both Ξ^- and Σ^- are isospin unfavored) [4]. The threshold condition for baryon B is

$$\mu_n \geqslant q_B \mu_e + g_{\omega B}\omega_0 + g_{\rho B}\rho_{03} I_{3B} + m_B - g_{\sigma B}. \qquad (11)$$

The sign of $g_{\rho B}\rho_{03}$ is determined by the net isospin density of the star, which is dominated by the neutron. The first term on the left determines whether a given baryon charge state is charge favored or unfavored and the third term whether it is isospin favored or unfavored.

The maximum neutron star mass is only somewhat perturbed by uncertainty in the Σ^- coupling as can be seen in Fig. 4. It is seen that hyperons significantly reduce the limiting neutron star mass to a value $\sim 1.5 M_\odot$ in this theory with coupling constants chosen in accordance with nuclear and hypernuclear data. The latter data are not nearly as firm as the former and introduce some uncertainty. A limit of $\sim 1.7 M_\odot$ for neutron stars would be compatible with these uncertainties, but is in our estimation less favored than the first limit mentioned [1]. Either of these limits leaves a very small mass window for the existence of neutron stars, for the occurrence of supernovas (powered as they are by the neutron star binding energy), and therefore for a universe containing heavy elements and at least one planet with life (2nd edition of [17]).

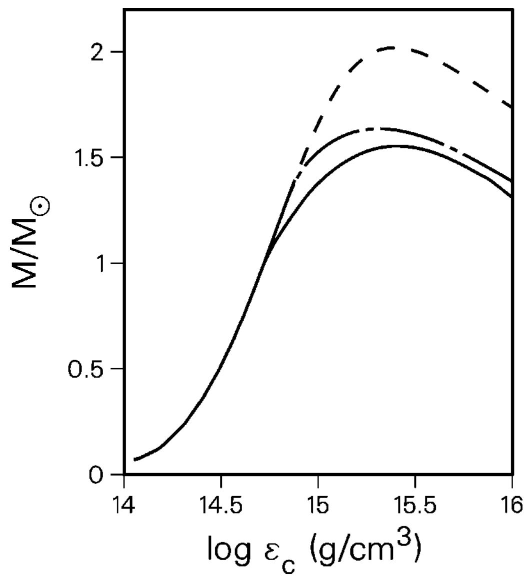

FIG. 4. Neutron star sequences corresponding to the three cases defined in Fig. 1. (Logarithm is to the base 10.)

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